Acoustic streaming at ultrasound resonances in microfluidic chambers: theory and simulation

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1. Introduction
In a series of recent papers written in collaboration with our experimental colleagues at the Technical University of Denmark (Jörg Kutter's group), at Lund University (Thomas Laurell's group) and at the Royal Institute of Technology (Martin Wiklund's group), we (Henrik Bruus's group) have demonstrated how theoretical analysis and full numerical simulation play an important role for the interpretation of various measurements of acoustic radiation forces and acoustic streaming in microsystems exposed to piezo-induced ultrasound tuned to resonance conditions [1–3].

2. Setup
The typical setup is shown in Figs. 1(a) and (b). A silicon-based microfluidic system, in which a water-filled chamber is connected to in- and outlets and sealed by a glass lid, is placed on top of a piezo crystal. By properly tuning the MHz frequency of the AC-voltage bias applied to the piezo crystal, ultrasonic resonances can be set up in the microchamber. A numerical simulation of the first-order pressure field $p_1$ at one particular resonance in an arbitrarily shaped micro chamber is shown as the color-scale plot in Fig. 1(c), where also the pressure nodal lines are indicated by thin black lines.

3. Governing equations
A basic introduction to the theory of acoustofluidics is given in Ref. [4]. The first-order pressure and velocity fields, $p_1$ and $v_1$, are found through the Helmholtz equation for the entire system given a time-harmonic excitation $\cos(\omega t)$ modeling the action of the piezo crystal. The small viscous damping is not taken into account in this part of the calculation. The resulting first-order fields then act as sources to the second-order acoustic pressure and velocity fields, $p_2$ and $v_2$. Due to the inherent non-linearity of the Navier–Stokes equation, the second-order fields have non-zero time averages even for the harmonic temporal first-order excitation. It is these slowly varying time averaged responses that are observed in particle image velocimetry measurements [1].
4. Acoustic streaming and loss of acoustic energy through the chamber walls

Our analysis indicates that the observed acoustic streaming, depicted in Fig. 2(a) adapted from Ref. [1], with its $6 \times 6$ vortex pattern, can only be understood in terms of the first-order acoustic pressure and velocity fields, $p_1$ and $v_1$, shown in Fig. 2(b), if the loss of first-order acoustic radiation energy through the walls of the chamber is taken properly into account, see Figs. 2(c)–(f).

![Fig. 2](http://www.ucl.ac.uk/medicine/hepatology-rf/research/usw-net/)

Fig. 2. (a) Acoustic streaming in a $2 \times 2 \times 0.2$ mm$^3$ chamber measured by micro particle image velocimetry, adapted from Ref. [1]. (b) Simulated first-order pressure field $p_1$ in color scale. (c)–(f) Simulated bulk part $\langle v_2 \rangle = -\langle p_1 v_1 \rangle / (\rho_0 c_a^2)$ of the acoustic streaming with various configurations of hard wall boundary conditions (full lines) and lossy acoustic impedance boundary conditions (dashed lines). Note how the asymmetric $6 \times 6$ vortex patterns in panel (c)–(e) changes into the symmetric $6 \times 6$ vortex pattern in panel (f) as the boundary conditions are changed from being asymmetric to symmetric.

References


